Figure 1.5

Sample Unit Plan Progression of Content for Applying Properties of Operations as Strategies to Multiply for Grade 3

<table>
<tr><td>Unit Name: Multiplication Facts and Strategies</td><td>Unit Number: 4</td></tr>
<tr><td>Standard
3.OA.B.5: Apply properties of operations as strategies to multiply and divide.* Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known (commutative property of multiplication). $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$ (associative property of multiplication). Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (distributive property).</td><td>Mathematical Practices
• Mathematical Practice 3: “Construct viable arguments and critique the reasoning of others.”
• Mathematical Practice 4: “Model with mathematics.”
• Mathematical Practice 7: “Look for and make use of structure.”
• Mathematical Practice 8: “Look for and express regularity in repeated reasoning.”</td></tr>
<tr><td>Time Frame
4–1 to 4–13</td><td>Purpose
This unit develops the concept of using strategies based on properties of operations (such as the commutative, associative, and distributive properties) to multiply basic facts.
Strategies to focus on:
• Doubling strategy
• Changing order of factors
• Break-apart strategy</td></tr>
<tr><td>Overview of the Essential Learning Standards
I can use the doubling strategy to multiply.
I can change the order of factors to use facts I know to multiply.
I can use the break-apart strategy to multiply.
I can represent multiplication strategies with equations.</td><td>Enduring Understanding and Essential Questions
How can you choose appropriate strategies based on properties of operations to multiply?</td></tr>
<tr><td colspan="2">Prior Knowledge (What Knowledge and Skills Need to Be Spiraled?)
• Prerequisites for this unit are:
• Represent multiplication as groups of objects
• Represent multiplication with concrete objects
• Connect multiplication to repeated addition</td></tr>
<tr><td colspan="2">Key Vocabulary
• array • factor • product
• even • odd • equation</td></tr>
</table>

Assessment Evidence

- Mathematics journal prompts
- Classroom discourse
- Independent practice
- + Assessment 4–7: Using doubling and the commutative property
- + Assessment 4–13: Using doubling, the commutative property, and the break-apart strategy

Learning Plan (Unit Description)	Suggested Teaching Strategies, Procedures, and Notes to Teachers (For example, How will prior knowledge be addressed? When will assessments be used?)
Day 1	**Topic:** Multiply with 2. **Doubling facts:** Solve problems with a 2 as the first factor by doubling, such as 2×6 by thinking $6 + 6$. **Mathematical Practice 8:** Students explore many examples of facts with 2 as the first factor and notice that in each case, they can solve the facts by adding the second factor to itself, thus doubling the second factor. **Essential learning standard 1:** I can use the doubling strategy to multiply by 2. (3.OA.5)
Day 2	**Topic:** Multiply with 4. **Doubling doubles:** Solve problems with 4 as the first factor by first solving the double and then multiplying that product by 2. For example, 4×6 becomes $2 \times (2 \times 6)$. Students think 2×6 then double. **Mathematical Practice 3:** Students are challenged to apply the doubling strategy to facts with 4 as the first factor. The goal is for students to conjecture that they are doubling a double. Students must express their thinking and make sense of the thinking of others. **Essential learning standard 1:** I can multiply by 4 by doubling the product of multiplying by 2. (3.OA.5)
Days 3 and 4	**Topic:** Use doubles to multiply with 6 and 8. **Extending doubles:** Extend doubling strategy to all facts with an even first factor. For example, 6×7 becomes $2 \times (3 \times 7)$. Make the connection to the associative property explicit. **Mathematical Practice 7:** Students see that all even numbers can be described as $2 \times n$. Once they identify this structure, students apply the doubling strategy to all facts with even first factors. **Essential learning standard 1:** I can use the doubling strategy to multiply by even factors. (3.OA.5)
Day 5	**Topic:** Model with arrays. **Modeling the commutative property:** Prepare for making sense of the commutative property by modeling products with arrays. **Mathematical Practice 8:** Students use arrays to represent several facts and see that facts in the form of $a \times b$ have the same product as facts in the form of $b \times a$. Students see that the columns and rows are just interchanged, leaving the same number of total objects in the array. **Essential learning standard 2:** I can use an array to represent multiplication. (3.OA.5)

Day 6	**Topic:** Use the commutative property. **Using the commutative property:** Make sense of the commutative property through the use of arrays so that students can use the doubling strategy with factors that have an odd first factor and an even second factor. For example, $7 \times 6 = 6 \times 7$, so the fact can be solved using the doubling strategy. **Mathematical Practice 8:** Students use arrays to represent several facts and see that facts in the form of $a \times b$ have the same product as facts in the form of $b \times a$. They see that the columns and rows are just interchanged, leaving the same number of total objects in the array. **Essential learning standard 2:** I can change the order of factors to use facts I know to multiply.
Day 7	**Assessment 4–7:** Use doubling and the commutative property.
Day 8	**Topic:** Multiply with 5. Apply the commutative property and skip counting to multiply with 5. **Essential learning standard 2:** I can change the order of factors and use skip counting to multiply with 5.
Day 9	**Topic:** Break apart numbers to multiply. **Using the distributive property:** Derive the product of facts from known facts by using the break-apart strategy. For example, solve 6×7 by using arrays to see that you can solve 6×7 by thinking 5×7 then add 7. **Mathematical Practice 7:** Students use arrays to see that they can find parts of arrays and combine them to determine the total number of objects in the arrays. **Essential learning standard 3:** I can use the break-apart strategy with facts I know to multiply.
Day 10	**Topic:** Multiply with 3 and 6. Apply the break-apart strategy and the commutative strategy to find products of facts with factors of 3 or 6. **Mathematical Practice 7:** Students use arrays to see that they can find parts of arrays and combine them to determine the total number of objects in the arrays. **Essential learning standard 3:** I can use the break-apart strategy and change the order of factors to multiply.
Day 11	**Topic:** Multiply with 7 and 9. Apply the break-apart strategy and the commutative strategy to find products of facts with factors of 3 or 6. **Mathematical Practice 7:** Students use arrays to see that they can find parts of arrays and combine them to determine the total number of objects in the arrays. **Essential learning standard 3:** I can use the break-apart strategy with facts I know to multiply.

Day 12	**Topic:** Use the associative and distributive properties (algebra). Connect the break-apart strategy to the distributive property by showing that 6×7 is really $(5 + 1) \times 7$ and equals $(5 \times 7) + (1 \times 7)$. **Mathematical Practice 4:** Students use the array they explored with the break-apart strategy to write expressions to represent the partial products. **Essential learning standard 4:** I can represent multiplication strategies with equations.
Day 13	**Assessment 4–13:** Use doubling, the commutative property, and the break-apart strategy.

Differentiation	
Enhancement	**Remediation**
Students write word problems to match a given expression and then apply strategies to multiply. Students explore multiple strategies for the same fact, providing justification for which strategy is most efficient.	Students use concrete materials to represent properties of operations. Students apply properties of operations as strategies using facts with lesser factors.
Resources	**Technology**
• Two-color counters • Grid paper	None

**While some Mathematical Practices are pervasive throughout the unit, such as Mathematical Practice 3, it is important to target specific practices for planning purposes.*

Source for standards: National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for mathematics. *Washington, DC: Authors. Accessed at www.corestandards.org/assets/CCSSI_Math%20Standards.pdf on February 7, 2014. (See pp. 6–8, 23.)*