APPENDIX E

Changes in Mathematics Standards, 1989–2012

Helping students use their prior knowledge to enable them to recognize what is new and different in their learning is a key element of scaffolded instruction. Similarly, as you explore the CCSS for mathematics, it will be helpful to compare aspects of mathematics standards that have framed your previous instruction so that you can identify what is familiar, what is new and challenging, and what changes are required in the content delivered to your students. As you examine the CCSS for mathematics, you may find it helpful to refer to the standards that have formed the basis of your instruction recently. The release of the CCSS for mathematics comes on the heels of a long history of mathematics research and reform efforts. Consider the following dates.

- **1957:** Launch of *Sputnik* resulted in the "new math" movement, with an emphasis on the abstract nature of mathematical structure, set theory, and number bases
- **1970s:** Back-to-basics movement in which rote memorization was emphasized—the teacher was the dominant figure in instruction with good management skills and a focus on basic skills.
- **1983:** *A Nation at Risk* brought education into the spotlight and included recommendations for three years of high school mathematics. This document was critical of the back-to-basics movement and supported NCTM's 1980 policy statement An Agenda for Action.
- **1989:** NCTM publishes *Curriculum and Evaluation Standards for School Mathematics*, emphasizing more problem solving and reasoning of mathematics.
- **1991:** NCTM publishes *Professional Standards for Teaching Mathematics*.
- **1995:** NCTM publishes *Assessment Standards for School Mathematics*.
- **2000:** NCTM publishes *Principles and Standards for School Mathematics* (PSSM) providing a vision for teaching and learning mathematics, including the Process Standards (problem solving, reasoning and proof, communication, representation, and connections).
- **2001:** In *Adding It Up: Helping Children Learn Mathematics*, the National Research Council identifies five strands that comprise mathematical proficiency (adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition).

- **2006, 2009:** NCTM releases *Curriculum Focal Points for Prekindergarten Through Grade 8: A Quest for Coherence* and *Focus on High School Mathematics: Reasoning and Sense Making*, two documents that further develop the conceptual ideas captured in PSSM.
- **2010:** The CCSS for mathematics are released, and forty-five state governments adopt them.

What becomes readily apparent in this brief history is that content standards are not new for you. Research confirms the notion that learning mathematics goes beyond a demonstration of knowledge of the content and must also include the ways in which students reason and make sense of the mathematics they are learning.

The placement of content standards and mathematical processes is an important consideration in any set of standards. The content and process standards within the *Principles and Standards for School Mathematics* (NCTM, 2000) were the same across all the grade-level bands (preK–2, 3–5, 6–8, and 9–12) and are most likely familiar to you.

The Common Core State Standards (NGA & CCSSO, 2010a) differ significantly from the *Principles and Standards for School Mathematics* (NCTM, 2000) in the descriptive language used to define the content standards. The CCSS reference content areas as *domains* rather than *content topics* or *strands*. Similarly, the content domains within the CCSS differ according to level rather than being the same from preK–12. These differences are illustrated in table E.1, which shows content topics defined in the *Principles and Standards for School Mathematics* and the content domains defined in the Common Core State Standards. You can use this table with your collaborative learning team to focus discussion on these three questions:

1. How familiar are the terms describing content topics and content domains?
2. How does the depth and coverage of content compare to what you have followed in the past?
3. What are your teacher professional development needs and differences between the *Principles and Standards for School Mathematics* and the Common Core State Standards content?

In 2006, NCTM released the *Curriculum Focal Points*. The *Curriculum Focal Points* were intended to serve as a discussion document for states, school districts, and local schools as they began a conversation around the more important or focus topics at particular grades for levels K–8. Many states saw the Focal Points as an opportunity for their schools or school districts to identify areas of curricular focus within particular grades and also to provide the grade-by-grade essentials for all students. The Focal Points became one of the foundational guides to the Common Core State Standards. One aspect of the CCSS is the critical areas presented at the beginning of each grade level's discussion (see appendix B, page 185, for example). The critical areas are grade-level emphasis points and are, in essence, the *Curriculum Focal Points*, with some revision. The

Table E.1: Mathematics Content—*Principles and Standards for School Mathematics* and the Common Core State Standards

PSSM—Content Topics Grades PreK–12	CCSS—Content Domains Grades K–5	CCSS—Content Domains Grades 6–8	CCSS—Conceptual Categories High School
Number and Operations	Counting and Cardinality (K only)	Ratios and Proportional Relationships (Grades 6–7)	Number and Quantity
	Number and Operations in Base Ten	The Number System	
	Number and Operations–Fractions (Grades 3–5 only)		
Algebra	Operations and Algebraic Thinking	Expressions and Equations	Algebra
Geometry	Geometry	Geometry	Geometry
Measurement		Functions (Grade 8 only)	Functions
Data Analysis and Probability	Measurement and Data	Statistics and Probability	Statistics and Probability
			Modeling

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critical areas for grades 6–8 are shown in table E.2 (page 4 of this appendix), which provides a second look at the important content across these grades. You can use this table to extend the discussion started with a review of table 3.1 (page 79) with your collaborative learning team. As your collaborative team begins work analyzing the content focus across the grades, you can draw on these questions to frame the discussion.

1. How much time do you think should be allotted for each critical area? How will these changes in time allocation be accommodated in building and classroom scheduling?
2. How are the critical areas similar to or different from topics you have emphasized in your teaching now or in the past?
3. What impact will work from the prior year have on the topics specified in the critical areas?

Table E.2: CCSS Critical Areas and NCTM Focal Points, Grades 6–8

Grade Level	CCSS Critical Areas	NCTM Focal Points
Grade 6	1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems. 2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative number. 3. Writing, interpreting, and using expressions and equations.	1. Developing an understanding of and fluency with multiplication and division of fractions and decimals 2. Connecting ratio and rate to multiplication and division 3. Writing, interpreting, and using mathematical expressions and equations
Grade 7	1. Developing understanding of and applying proportional relationships. 2. Developing understanding of operations with rational numbers and working with expressions and linear equations. 3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume. 4. Drawing inferences about populations based on samples.	1. Developing an understanding of and applying proportionality, including similarity 2. Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes 3. Developing an understanding of operations on all rational numbers and solving linear equations
Grade 8	1. Formulating and reasoning about expressions and equations, including modeling an ass0ciation in bivariate data with a linear equations, and solving linear equations and systems of linear equations. 2. Grasping the concept of function and using functions to describe quantitative relationships. 3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.	1. Analyzing and representing linear functions and solving linear equations and systems of linear equations 2. Analyzing two- and three-dimensional space and figures by using distance and angle 3. Analyzing and summarizing data sets

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The Standards for Mathematical Practice described what your students are doing as they engage in learning the CCSS mathematics content standards. Forerunners of these standards are found in the NCTM Process Standards (NCTM, 1991) and the NRC Strands of Mathematical Proficiency (NRC, 2001). Table E.3 shows the relationships among these standards.

Table E.3: CCSS Standards for Mathematical Practice, NCTM Process Standards, and NRC Strands of Mathematical Proficiency.

The CCSS Standards for Mathematical Practice	NCTM Process Standards	Strands of Mathematical Proficiency
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**Problem Solving** • Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and in other contexts • Apply and adapt a variety of appropriate strategies to solve problems • Monitor and reflect on the process of mathematical problem solving **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics • Make and investigate mathematical conjectures • Develop and evaluate mathematical arguments and proofs • Select and use various types of reasoning and methods of proof **Communication** • Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others	**Conceptual understanding:** Comprehension of mathematical concepts, operations, and relations **Procedural fluency:** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately **Strategic competence:** Ability to formulate, represent, and solve mathematical problems **Adaptive reasoning:** Capacity for logical thought, reflection, explanation, and justification **Productive disposition:** Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

continued →

The CCSS Standards for Mathematical Practice	NCTM Process Standards	Strands of Mathematical Proficiency
	• Analyze and evaluate the mathematical thinking and strategies of others; • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics **Representation** • Create and use representations to organize, record, and communicate mathematical ideas • Select, apply, and translate among mathematical representations to solve problems • Use representations to model and interpret physical, social, and mathematical phenomena	

The National Mathematics Advisory Panel (2008) identifies the Critical Foundations of Algebra. These clusters of concepts and skills are essentials for all students prior to formal coursework in algebra and include the following major content topics, with suggested grade-level benchmarks for grades 6–8.

1. **Fluency with whole numbers:**
    a. By the end of grade 5, students should be proficient with multiplication and division of whole numbers.

2. **Fluency with fractions:**
    a. By the end of grade 6, students should be proficient with multiplication and division of fractions and decimals.
    b. By the end of grade 7, students should be proficient with all operations involving positive and negative fractions.
    c. By the end of grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.
3. **Particular aspects of geometry and measurement:**
    a. By the end of grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
    b. By the end of grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

Discussing the critical areas of the CCSS and the NCTM's Focal Points (see table E.2, page 4 of this appendix), the relationships among the CCSS Standards for Mathematical Practice and the NCTM Process Standards and the NRC Strands of Mathematical Proficiency, as well as the NMAP's Critical Foundations, will build on your teams' initial examination of the general mathematics content of the CCSS. This subsequent discussion of their similarities and how they outline points and emphasize topics within the CCSS provides a second-level professional development opportunity for collaborative teams on the important mathematics for grades 6–8. This should prepare your collaborative teams to engage in meaningful discussions about what's mathematically important across these three grade levels. Additionally, these historical documents provide a precedent for curricular standards and eliminate major surprises regarding actual content topics when experienced teachers review the CCSS domains, standards, and clusters for grades 6–8.

When discussing the standards over the next few years, collaborative teams will frequently ask, "What does that really mean?" in regard to a standard or content standard cluster. The assessment instruments or tasks you choose to validate a standard will become the operational definition of that standard—it's what you expect students to do based on your understanding of the standard. Monitoring the progress of the two assessment consortia will be an important way for you to anticipate the next steps in providing the necessary focus for and connections to the CCSS content as well as the student behaviors described in the Standards for Mathematical Practice.