Big Ideas in Early Mathematics

Use this chart to identify specific big ideas in the lesson you are teaching. For later grades, identify the big ideas students need. There are far fewer of these than there are standards, but they are often the building blocks for students to master more standards. Whether or not these big ideas are specifically assessed on accountability tests, they are essential to students' being able to master more and more complex ideas in mathematics. These ideas often spiral, or instruction loops back, with a new, deeper level of understanding. They are thus listed in alphabetical order rather than an order for mastery.

Associative property: An operation is associative if you can group the numbers in any way without changing the answer.

- Work with fifteen counters. Ask a student, "Could you arrange them so that you have a group of eight and a group of seven? How many are there? Now make groups of eleven and four. How many markers do you have?" If they recount after rearranging, ask them why they did that.
- Use related equations in number talks, leaving the answer to the first equation on the board so that students who remember the associative property can use it as a strategy and help the rest of the class learn it. For example:
    - **Problem 1:** (2 x 3) x 5 (mental math)
    - **Problem 2:** 2 x (3 x 5) (mental math)

Cardinality: Cardinality is an understanding that the name of a number relates to a specific quantity of objects. "How many?"

- Ask a student to hold up three fingers. Ask the student, "Where's the three?" If the student points to the last finger, rather than to the three total fingers, the student is still thinking of numbers as names, not quantities.
- Use matching games, such as matching numbers and the numbers of objects in a picture.

Combinations that make five and ten: This refers to the use of automaticity with combinations of numbers that make five and ten.

- Talk about using these combinations. "We know that 4 + 1 is 5. So, to add 4 and 2, we can apply that combination of 5: 5 + 1 = 6. Who can draw this for the class?"
- Play the memory game (also known as pairs or concentration) with just cards for numbers one to five or for numbers one to nine. Students keep cards when they draw a pair that adds to five or ten.

Commutative property: An operation is commutative if you can change the order of the numbers involved without changing the value of the equation.

- Use triangle fact cards (both commercial sets and reproducible masters are widely available) where, for example, 5, 3, and 8 are placed in the points. Note that these cards also embed the relationships between addition and subtraction and between multiplication and division, demystifying the latter operations.
- Use number talks where changing the order of numbers makes a problem easier to solve. (The problem 8 + 3 + 2 is one example.) When students use the commutative property, name it, or ask them, "What's that called again?"

Compensation: In addition and subtraction, if you add to or subtract from one addend (or subtrahend) to make a friendlier number, that quantity must be subtracted from or added to the other addend (or subtrahend) to maintain equivalency, such as 8 + 6 = (8 + 2) + (6 – 2) = 10 + 4.

- Ask questions such as, "I'd like you to add 8 and 3, but is there a way you can use one of our friendly numbers to do that? Why does that work?"
- Practice mathematics facts in families that demonstrate compensation (such as 5 + 1, 4 + 2, and so on, or 75 + 28, 74 + 29, and so on).

Conservation: The number of objects remains the same no matter how the objects are arranged.

- Line up twelve counters in two rows of six. Ask a student how many counters there are. Then ask the student to place the counters in four equal rows. Ask how many counters there are now. What if they're in one row? How many are there?

- Find different trays that students can count objects into—egg cartons, mancala boards, muffin tins, strips of paper with six boxes marked off, and so on. Number the bottom of each tray with places for numbers 1–6. Have them drop six counters, one by one, into each tray. Does each tray have six? How can they tell? Does six look the same in each one?

Counting on: This is the ability to continue in counting, as opposed to starting from zero, when adding more to a sum.

- Place six counters in front of a student, and ask the student to count them. Add three more, and ask the student, "Now how many are there?" The student understands counting on if the student simply says, "Seven, eight, nine," rather than starting over.
- Play "Captain, May I?" with counting. Students line up at the starting line. Teacher lines might be, "Take three steps. Who can tell me how many steps you'll have taken in total if you add two more steps?"

Hierarchical inclusion: Numbers build exactly one by one, and they nest within each other by this amount (five, four, three, two, and one nest in six).

- Using counters, ask, "If I add one more, how many counters will I have? If I take one away, how many will I have?" With the same counters, emphasize, "How many do I need to make five? Ten?" to demonstrate combinations of five and ten as well as how the numbers nest the combinations.
- Play 7 Ate 9 (a commercial card game). You might start with the simplest cards in the deck.

Magnitude: This means understanding, without counting, which group of objects has more parts (an easier concept to grasp than cardinality).

- Show student groups two sets of objects side by side, one with five objects and one with seven objects. Make sure that one group's objects aren't spread so far apart that the students might be misled. Ask, "Without counting, can you tell which is the bigger group? How do you know?"
- Lay out between one and seven counters. Ask students to make a group of counters that is larger (or smaller) than your group. Or, draw a number of circles (faces) and, separately, set out a number of buttons for eyes. Ask students whether they think there are enough eyes for the faces and how they can find out. Repeat this activity for mouths, noses, and so on. Speak in terms of more than, less than, as many as, and How can we find out?

One-to-one correspondence: Counting means one number per object.

- Make counting part of class—counting people on the cover of a book, trees in a picture, dice in a bag, and so on.
- Show a row of five grapefruit and a row of five oranges (or show rows of two other objects that are similar yet different in size), and ask whether there is the same number of objects in each row.

Subitizing: This involves immediately recognizing a collection of two, three, or four objects as a single unit.

- Using dot cards and five frames, check whether a student can recognize the quantities without having to count each dot.
- Play memory games with cards that feature one, two, three, and four objects, encouraging students to practice letting their eyes recognize how many objects there are rather than counting each object.

Unitizing: This refers to the ability to understand the units a numeral represents. For example, the number 2 always represents two units, but it means two tens in 21 and two ones in 12.

- Ask questions such as, "In 1,624, the one is worth one thousand. What is the two worth?" and "What is ten less than 4,000?"
- Utilize quick whiteboard problems such as, "Write a three-digit number where the two is worth twenty and one where it is worth two thousand." Have students place their numbers on small whiteboards at the front of the room, or compare their numbers in small groups, and work together to see whether all their answers are correct.