All Mathematics Moves

Please see the following tables to view all mathematics mentor texts pertaining to each mini move.

Moves That Introduce: Mathematics

Move	Mini-Mentor Text
Just-the-Facts	In the 1950s, four decades before he won a Nobel Prize for his contributions to game theory and his story inspired the book and film "A Beautiful Mind," the mathematician John Nash proved one of the most remarkable results in all of geometry. Among other features, it implied that you could crumple a sphere down to a ball of any size without ever creasing it. He made this possible by inventing a new type of geometric object called an "embedding," which situates a shape inside a larger space—not unlike fitting a two-dimensional poster into a three-dimensional tube. —Mordechai Rorvig (2021), "Mathematicians Identify Threshold at Which Shapes Give Way," *Quanta Magazine*
Make the Case	In a new proof, a long-neglected mathematical object has finally gotten its moment in the spotlight. At first glance, modular forms—functions whose abundant symmetries have intrigued mathematicians for centuries—seem to have garnered more than enough attention. They crop up in all sorts of problems: They were a key ingredient in Andrew Wiles' 1994 proof of Fermat's Last Theorem, which resolved one of the biggest open questions in number theory. They play a central role in the Langlands program, an ongoing effort to develop "a grand unified theory of mathematics." They've even been used to study models in string theory and quantum physics. —Jordana Cepelewicz (2023b), "New Proof Distinguishes Mysterious and Powerful 'Modular Forms,'" *Quanta Magazine*
What They Said	"For me, mathematics exists in the space between us," Emmy Murphy wrote in accepting the 2020 New Horizons in Mathematics Prize. That space, for her, is a realm of art, perhaps even more than science. And like an artist, she is most fulfilled when exploring the fertile ground where constraint meets creation. The objects she studies are "beautiful to me in the same way that architecture or fashion or expensive furniture is beautiful—the way they are both highly constrained by their geometry and also highly flexible," she told *Quanta*. —Erica Klarreich (2023a), "Emmy Murphy Is a Mathematician Who Finds Beauty in Flexibility," *Quanta Magazine*
Scene-Drop	A few minutes into a 2018 talk at the University of Michigan, Ian Tobasco picked up a large piece of paper and crumpled it into a seemingly disordered ball of chaos. He held it up for the audience to see, squeezed it for good measure, then spread it out again. "I get a wild mass of folds that emerge, and that's the puzzle," he said. "What selects this pattern from another, more orderly pattern?" He then held up a second large piece of paper—this one pre-folded into a famous origami pattern of parallelograms known as the Miura-ori—and pressed it flat. The force he used on each sheet of paper was about the same, he said, but the outcomes couldn't have been more different. The Miura-ori was divided neatly into geometric regions; the crumpled ball was a mess of jagged lines. —Stephen Ornes (2022), "The New Math of Wrinkling," *Quanta Magazine*
Then-and-Now	In the fourth century, the Greek mathematician Pappus of Alexandria praised bees for their "geometrical forethought." The hexagonal structure of their honeycomb seemed like the optimal way to partition two-dimensional space into cells of equal area and minimal perimeter—allowing the insects to cut down on how much wax they needed to produce, and to spend less time and energy building their hive. Or so Pappus and others hypothesized. For millennia, nobody could prove that hexagons were optimal—until finally, in 1999, the mathematician Thomas Hales showed that no other shape could do better. Today, mathematicians still don't know which shapes can tile three or more dimensions with the smallest possible surface area. —Jordana Cepelewicz (2023a), "Mathematicians Complete Quest to Build 'Spherical Cubes,'" *Quanta Magazine*

Moves That Make a Claim: Mathematics

Move	Mini-Mentor Text
The Big Idea	In a 912-page paper posted online on May 30, Szeftel, Elena Giorgi of Columbia University and Sergiu Klainerman of Princeton University have proved that slowly rotating Kerr black holes are indeed stable. —Steve Nadis (2022a), “At Long Last, Mathematical Proof That Black Holes Are Stable,” *Quanta Magazine*
Outline It	Four Fields Medals were awarded for major breakthroughs in geometry, combinatorics, statistical physics and number theory, even as mathematicians continued to wrestle with how computers are changing the discipline. —Konstantin Kakaes (2022), “The Year in Math,” *Quanta Magazine*
This-and-That	But it turns out that shapes with nonnegative Ricci curvature are more flexible and less well behaved than mathematicians had expected—complicating their understanding of the relationship between local geometric properties and global topological ones. —Jordana Cepelewicz (2024b), “Strangely Curved Shapes Break 50-Year-Old Geometry Conjecture,” *Quanta Magazine*
Not-This-But-That	General relativity has transformed our understanding of gravity, depicting it not as an attractive force between massive objects, as had long been held, but rather as a consequence of the way space and time curve in the presence of mass and energy. —Steve Nadis (2022b), “Mass and Angular Momentum, Left Ambiguous by Einstein, Get Defined,” *Quanta Magazine*
Synthesize It	What we do know is that the mathematical technique now known as the Chinese remainder theorem was devised sometime between the third and fifth centuries CE by the Chinese mathematician Sun Tzu (not to be confused with Sun Tzu who wrote *The Art of War* almost 1,000 years earlier). . . . . . . Sun Tzu never proved this formally, but later the Indian mathematician and astronomer Aryabhata developed a process for solving any given instance of the theorem. —Lakshmi Chandrasekaran (2021), “How Ancient War Trickery Is Alive in Math Today,” *Quanta Magazine*

Moves That Define: Mathematics

Move	Mini-Mentor Text
It Is What It Is	To formalize how close to a square a rectangle is, mathematicians use a number called the aspect ratio. It is simply the length divided by the width. —Kevin Hartnett (2024a), “Mathematicians Identify the Best Versions of Iconic Shapes,” *Quanta Magazine*
Say My Name	This process, called the unfolding of the billiard path, allows the ball to continue in a straight-line trajectory. —David S. Richeson (2024), “Unfolding the Mysteries of Polygonal Billiards,” *Quanta Magazine*
Keep It Appositive	Alongside this came the work of Christopher Clavius—a German Jesuit astronomer who helped Pope Gregory XIII to introduce the Gregorian calendar—and other mathematicians on fractions. —Madeleine S. Killacky (2023), “Shakespeare by Numbers: How Mathematical Breakthroughs Influenced the Bard’s Plays,” *The Conversation*
Gimme an Example	In general, a set is a group if it comes with some operation that, like addition, combines two elements into a third element in a way that satisfies some basic requirements. —Leila Sloman (2023), “Probability and Number Theory Collide—in a Moment,” *Quanta Magazine*
Engage With Etymology	The term *tally* comes from the French verb *tailler*, “to cut,” like the English word *tailor*; the root is seen in the Latin *taliare*, meaning “to cut.” It is also interesting to note that the English word *write* can be traced to the Anglo-Saxon *writan*, “to scratch,” or “to notch.” . . . Counts were maintained by making scratches on stones, by cutting notches in wooden sticks or pieces of bone, or by tying knots in strings of different colors or lengths. —David M. Burton (2011), *The History of Mathematics: An Introduction*, p. 2

Moves That Describe: Mathematics

Move	Mini-Mentor Text
Describing Lists	Wantzel used his results to resolve other classical problems by proving that they can't be solved—it is impossible to trisect some angles, it is impossible to double the cube and it is impossible to construct certain regular polygons. —David S. Richeson (2023b), "How Math Achieved Transcendence," *Quanta Magazine*
Say It Again, But Make It Specific	Let's call $g(x)$ the new quadratic function we get when we replace x with $x + 4$. In other words, let $g(x) = f(x + 4)$. —Patrick Honner (2023c), "The Symmetry That Makes Solving Math Equations Easy," *Quanta Magazine*
Dash That Describes	Still, the impact on payments for undergraduates is relatively modest—an addition of about $2 a month on a $5,500 loan (the first-year maximum borrowing amount), with a standard 10-year repayment term. —Ann Carrns (2023), "Expect Interest Rates on Federal Student Loans to Rise," *The New York Times*
Let's Imagine . . .	For example, let's say we have two tiles, A and B, and two rules for putting them together: 1. Next to an A, on either side you can only place a B. 2. Next to a B, on either side you can only place an A. Can we tile the line with these tiles and these rules? Absolutely. Suppose we put an A down first. —Patrick Honner (2023a), "Math Patterns That Go On Forever but Never Repeat," *Quanta Magazine*
Figurative Language Comparison	These classical problems could go down in infamy as sirens whose songs lured mathematicians to crash on the rocky shores of impossibility. But I see them as muses who inspired generations of creative thinkers. —David S. Richeson (2020), "When Math Gets Impossibly Hard," *Quanta Magazine*

Moves That Provide Evidence: Mathematics

Move	Mini-Mentor Text
Hyperlink Layers	After the start of the pandemic, Yang-Hui He, a researcher at the London Institute for Mathematical Sciences, decided to take on some new challenges. He had been a physics major in college, and had gotten his doctorate from the Massachusetts Institute of Technology in mathematical physics. But he was increasingly interested in number theory, and given the increasing capabilities of artificial intelligence, he thought he'd try his hand at using AI as a tool for finding unexpected patterns in numbers. (He had already been using machine learning to classify Calabi-Yau manifolds, mathematical structures that are widely used in string theory.) —Lyndie Chiou (2024a), "Elliptic Curve 'Murmurations' Found With AI Take Flight," *Quanta Magazine*
Reference a Visual	This movie created by the Harvard-led team shows the execution of a 48-logical-qubit circuit, which they say is the most advanced circuit ever executed on a quantum computer. Groups of eight atomic qubits are first brought together and entangled into error-corrected logical qubit blocks, indicated by red ovals. These blocks are then entangled with each other to create a circuit with hundreds of logical gate operations. . . . At the end of the computation, lasers read out atoms' states: If an atom is in the state that is resonant with the illumination, the light is scattered, but if it's in the other state, there's no scattering. [*In this image, a grid-like pattern consisting of small green dots arranged in two distinct rectangular groups. The top group has a more extensive and sparse arrangement, while the bottom group appears denser with a more compact arrangement of dots. The background is light, allowing the green dots to stand out clearly.*] —Philip Ball (2024a), "The Best Qubits for Quantum Computing Might Just Be Atoms," *Quanta Magazine*
The Fold In	Convex pentagons that tile the plane were trickier to classify. Reinhardt discovered five families of such pentagons; 50 years later, Richard Kershner found three more. Then in 1975, Martin Gardner wrote about the problem for *Scientific American*, bringing it to the attention of professional and amateur mathematicians alike. One such amateur, a computer programmer named Richard James III, sent Gardner an example of a ninth family, asking, "Do you agree that Kershner missed this one?" He had. —David S. Richeson (2023a), "A Brief History of Tricky Mathematical Tiling," *Quanta Magazine*
Paraphrase It	**Quote from the article "Persistent Homology for Resource Coverage: A Case Study of Access to Polling Sites" by Abigail Hickok, Benjamin Jarman, Michael Johnson, Jiajie Luo, and Mason A. Porter (2024):** Although we have explored a specific case study (namely, the accessibility of polling sites), it is also relevant to conduct similar investigations for other resources, such as public parks, hospitals, vaccine distribution centers, grocery stores, Planned Parenthood clinics, and Department of Motor Vehicles (DMV) locations. (p. 498) **Mentor text:** Topology concerns itself with continuous shapes, and polling sites are at discrete locations. But in recent years, topologists have adapted their tools to work on discrete data by creating graphs of points connected by lines and then analyzing the properties of those graphs. Hickok said these techniques are useful not only for understanding the distribution of polling places but also for studying who has better access to hospitals, grocery stores and parks. —Lyndie Chiou (2024b), "Topologists Tackle the Trouble With Poll Placement," *Quanta Magazine*
End With Analysis	If we want to better predict the system's properties at various energy states, it helps to understand the system when it's in its least excited state, which scientists refer to as the ground state. "A lot of chemists, material scientists and quantum physicists are working on finding ground states," said Robert Huang, one of the new paper authors and a research scientist at Google Quantum AI. "It is known to be extremely hard." It's so hard that after more than a century of work, researchers still haven't found an effective computational approach to determining a system's ground state from first principles. Nor does there appear to be any way for a quantum computer to do it. Scientists have concluded that finding a system's ground state is hard for both classical and quantum computers. —Lakshmi Chandrasekaran (2024), "Physicists Finally Find a Problem That Only Quantum Computers Can Do," *Quanta Magazine*

Moves That Summarize: Mathematics

Move	Mini-Mentor Text
Define and Detail	Graph theory can be thought of as a branch of combinatorics—the mathematical study of counting. Counting what can happen with collections of nodes and edges is, in some sense, a special case of counting combinations more generally. The year ended with a landmark proof by four prominent mathematicians of a longstanding conjecture that relates combinatorics to the algebraic structure of sets. —Konstantin Kakaes (2023), "The Year in Math," *Quanta Magazine*
Pivot Synopsis **[INSERT QR CODE]**	In Christian culture, 6 has a rather dark meaning and 666 is even called the *"Number of the Beast"*. The number 666 appears several times in the Bible, and in other old scripts. The ancient writers probably didn't know that 666 is related to the golden ratio by $\phi = -2 \sin(666°) = -2 \cos(6{\times}6{\times}6°)$, but maybe they were aware of some of the following "scary" relationships. —Philipp Legner (2010), "Editorial: 60 Issues of *Eureka*," *Eureka*, p. 4
The Devil in the Details	In string theory, there is a duality symmetry known as T-duality. This is a fundamental ambiguity in the description of the space time background in which the string lives. If the space time has some specific properties (technically, it should possess an isometry and be compact so that its first homotopy class is nontrivial) then there will be two backgrounds that will be related to each other that in ordinary differential geometry will be inequivalent and yet will be indistinguishable from the point of view of the string. These pairs are known as T-duals. This duality is stringy in nature and leads on to the idea of stringy geometry that differs from our usual notion of geometry in that such ideas of T-duality get built in. —George Ellis (2012), "Multiverses and Observational Limits of Cosmology," *Eureka*, p. 69
Cause and Effect Sandwich	Computers and powerful calculators are so deeply integrated into today's science—indeed, into every aspect of today's world—that it can be difficult to imagine working without them. At the beginning of the Manhattan Project, however, the only "computers" available for the complex calculations necessary were teams of assistants using mechanical hand calculators. Scientists' wives at Los Alamos were enlisted, the work divided amongst them to maximize efficiency—one dedicated to adding, one to dividing, one to cubing, and so forth. This basic form of calculation worked well initially, but as calculations of neutron mean free paths, critical densities, and shockwave propagation grew ever more complex and time-intensive, it strained to produce results in a timely manner. Further, the scarcity of materials (especially radioactive materials) and scientific manpower [meant] that extensive experimentation was impossible, so theoretical calculations were the only way to test central elements of the bomb designs before the final Trinity test. Los Alamos needed either far more teams of woman "calculators" or some new way of handling extensive calculations. —Office of History and Heritage Resources (n.d.), "The Manhattan Project: An Interactive History"
Quote It to Me	Nash's equilibrium concept, which earned him a Nobel Prize in economics in 1994, offers a unified framework for understanding strategic behavior not only in economics but also in psychology, evolutionary biology and a host of other fields. Its influence on economic theory "is comparable to that of the discovery of the DNA double helix in the biological sciences," wrote Roger Myerson of the University of Chicago, another economics Nobelist. —Erica Klarreich (2017), "In Game Theory, No Clear Path to Equilibrium," *Quanta Magazine*

Moves That Contextualize: Mathematics

Move	Mini-Mentor Text
Let's Compare	Modular arithmetic is often called clock arithmetic, as a 12-hour clock counts in mod 12. This way of thinking about counting around a circle like on a clock suggests that it might be useful for our problem. —Alvin Choy (2023), "The Josephus Problem," *Chalkdust*
Double Date	In 1940, from a jailhouse in Rouen, France, André Weil wrote one of the most consequential letters of 20th-century mathematics. He was serving time for refusing to join the French army, and he filled his days in part by writing letters to his sister, Simone, an accomplished philosopher living in London. —Kevin Hartnett (2024b), "A Rosetta Stone for Mathematics," *Quanta Magazine*
Show Me the Data	The events here are dependent upon each other, as opposed to independent. In the realm of probability, dependency of events is very important. For example, coin tosses are always independent events. When tossing a fair coin, the probability of it landing on heads, given that it previously landed on heads 10 times in a row, is still 1/2. Even if it lands on heads 1000 times, the chance of it landing on heads on the 1001st toss is still 50%. —Madeleine Hall (2021), "On Conditional Probability: Cards, COVID, and *Crazy Rich Asians*," *Chalkdust*
Educated Inference	Without the telltale sign of an obviously misshapen district to go by, mathematicians have been developing increasingly powerful statistical methods for finding gerrymanders. These work by comparing a map to an ensemble of thousands or millions of possible maps. If the map results in noticeably more seats for Democrats or Republicans than would be expected from an average map, this is a sign that something fishy might have taken place. —Mike Orcutt (2023), "How Math Has Changed the Shape of Gerrymandering," *Quanta Magazine*
Past and Present Connection	Pierre de Fermat has his name on one of the most famous theorems in mathematics. For over 300 years, Fermat's Last Theorem stood as the ultimate symbol of unachievable mathematical greatness. In the 1600s, Fermat scribbled a note about his proposed theorem in a book he was reading, claiming to know how to prove it without providing any details. Mathematicians attempted to solve the problem themselves until the 1990s, when Andrew Wiles finally proved it using new techniques discovered hundreds of years after Fermat died. —Patrick Honner (2023b), "Pierre de Fermat's Link to a High School Student's Prime Math Proof," *Quanta Magazine*

Moves That Add Voice: Mathematics

Move	Mini-Mentor Text
Say It Slang	The three researchers were trying to show that there couldn't be too many of these points, but so far, all their techniques had come up short. They seemed to be spinning their wheels. —Erica Klarreich (2024), "Merging Fields, Mathematicians Go the Distance on Old Problem," *Quanta Magazine*
Ask a Question	Then Peluse had a thought: What if they ditched the harmonic analysis problem—temporarily, of course—and turned their attention to sets of points in which the distance between any two points is exactly an integer? What possible structures can such sets have? —Erica Klarreich (2024), "Merging Fields, Mathematicians Go the Distance on Old Problem," *Quanta Magazine*
Put It in Parentheses	The first proof of this (there would be several) is commonly attributed to Pythagoras, a 6th-century BCE philosopher, even though none of his writings survive and little is known about him. —Jordana Cepelewicz (2024a), "How the Square Root of 2 Became a Number," *Quanta Magazine*
Connect Personally	This is the "handshake problem," and it's one of my favorites. As a math teacher, I love it because there are so many different ways you can arrive at the solution, and the diversity and interconnectedness of those strategies beautifully illustrate the power of creative thinking in math. —Patrick Honner (2024), "Math That Connects Where We're Going to Where We've Been," *Quanta Magazine*
Make It Metaphorical	Just as the software of your laptop runs without having to keep track of all the microscale information about the electrons in the computer circuitry, so emergent phenomena are governed by macroscale rules that seem self-contained, without heed to what the component parts are doing. Using a mathematical formalism called computational mechanics, the researchers identified criteria for determining which systems have this kind of hierarchical structure. . . . They tested these criteria on several model systems known to display emergent-type phenomena, including neural networks and Game-of-Life-style cellular automata. —Philip Ball (2024c), "The New Math of How Large-Scale Order Emerges," *Quanta Magazine*

Moves That Conclude: Mathematics

Move	Mini-Mentor Text
What We Don't Know and What We Do	Meanwhile, another question looms: To create a black hole of three spatial dimensions, must an object be compressed in all three directions, as Thorne insisted, or could compression in two directions or even just one be enough? All evidence points to Thorne's statement being true, Khuri said, though it is not yet proved. Indeed, it is just one of many open questions that persist about black holes after they first manifested more than a century ago in a German soldier's notebook. —Steve Nadis (2023), "Math Proof Draws New Boundaries Around Black Hole Formation," *Quanta Magazine*
What's Next?	Socolar, as a physicist, has begun exploring the tilings' material properties. The diffraction pattern that emerges if you shine a light through one of these tilings, he has found, has the same kind of sharp peaks researchers have observed in quasicrystals. Even so, the hat tiling "looks to me to be different from anything else I've seen before," he said. Meanwhile, Smith isn't done with his "tricky little tile." He intends to explore its artistic possibilities and figure out how to use colors to bring out the patterns the tile appears to insist upon. —Erica Klarreich (2023b), "Hobbyist Finds Math's Elusive 'Einstein' Tile," *Quanta Magazine*
Share the Last Word	But he also pointed out that pseudorandom processes are powerful tools, and projects like constructing primes are just one way of using them to connect ideas from mathematics, computer science, information theory and other areas. "It's exciting to try and think where else these brilliant observations will lead," Tell said. —Stephen Ornes (2023), "How to Build a Big Prime Number," *Quanta Magazine*
The Bottom Line	Imai is an expert witness for the plaintiffs in the pending Supreme Court case. The plaintiffs are arguing, using the ensembles he generated, that Alabama's districts violate the Voting Rights Act by disenfranchising Black voters. But the state of Alabama, which is being sued, is using his ensembles to argue that the map is fairly drawn. Whatever the court decides, it shows that, when it comes to elections, mathematics will always need to contend with politics. —Mike Orcutt (2023), "How Math Has Changed the Shape of Gerrymandering," *Quanta Magazine*
Solve the Problem	Sailors facing an imminent storm don't have years to study the growing waves around them, but the new framework represents small steps toward the long-term goal for rogue wave prediction—machinery that scans the ocean and sounds the alarm when it's time to batten down the hatches. LDT tools, once they're modified for the open ocean, could be one way for those on boats like the München to know the moment a rogue wave starts focusing. —Charlie Wood (2020), "The Grand Unified Theory of Rogue Waves," *Quanta Magazine*

Moves That Organize: Mathematics

Move	Mini-Mentor Text
Topic Sentence Transition	The modern notation for square roots came into use in the 16th and 17th centuries. But still, there was something slippery about them. Does √2 exist in the same way that 2 does? It wasn't clear. Mathematicians continued to live with that ambiguity. Then, in the mid-1800s, Richard Dedekind, among others, realized that calculus—which had been developed 200 years earlier by Isaac Newton and Gottfried Leibniz—stood on a shaky foundation. —Jordana Cepelewicz (2024a), "How the Square Root of 2 Became a Number," *Quanta Magazine*
Hinge Transition	The mathematicians ended up with what they called a smooth fractal snowflake—an infinite and delicate self-similar structure. It had nonnegative Ricci curvature at every point. And it had an infinite number of holes. They had disproved Milnor's conjecture. "It's more complicated than all the previous constructions" of manifolds with nonnegative Ricci curvature, said Guofang Wei of the University of California, Santa Barbara. —Jordana Cepelewicz (2024b), "Strangely Curved Shapes Break 50-Year-Old Geometry Conjecture," *Quanta Magazine*
List It	For example, let's say we have two tiles, A and B, and two rules for putting them together: Next to an A, on either side you can only place a B. Next to a B, on either side you can only place an A. —Patrick Honner (2023a), "Math Patterns That Go On Forever but Never Repeat," *Quanta Magazine*
Add Subheadings	Subheadings: • *The big jump* • *Proof of concept* • *Soulful solutions* —Siobhan Roberts (2024), "A.I.'s Latest Challenge: The Math Olympics," *The New York Times*
Visual Anchoring	For example, you can use scissors congruence to compute the area of a pentagon. Since area is preserved if you cut the pentagon up into smaller triangles, you can instead find the area of these triangles (using "½ base × height") and add them up to get the answer. [*Illustration of a pentagon broken into five smaller triangles that are then added together*] —Maxine Calle and Mona Merling (2023), "A Brief Illustrated Guide to 'Scissors Congruence'—an Ancient Geometric Idea That's Still Fueling Cutting-Edge Mathematical Research," *The Conversation*